

Accurate Autonomous Robot Laser Navigation

Using Only Passive Reflectors

Richard T. Vannoy II, B.S.E.E., A.S.C.S.

ITT Technical Institute, San Diego, California

richardv2@home.com

www.trainprogrammers.com

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In preparation for a navigation contest hosted by the San Diego Robotics Society (www.sdrobotics.org), the question came up about the possibility of accurate robot navigation in a robot playing field without the use of IR, sonar or laser range finders. This paper describes the theory behind a bearings-only laser navigation system, and is divided into three sections:

Section 1: The Solution and the Mathematical Algorithm (This paper) This paper is primarily the mathematical algorithm used. However it is written so that a reader without a background in geometry and trigonometry should be able to follow the process. The algorithm requires some geometry; the use of the Inscribed Angle Rule in a circle and some trigonometry such as the basic trigonometry functions and the Law of Cosines.

Section 2: Pseudocode for a Computer Program Derived from the Mathematical Algorithm (Future paper) This section will allow readers with some software

competence to produce code in any language that will comply with the algorithm.

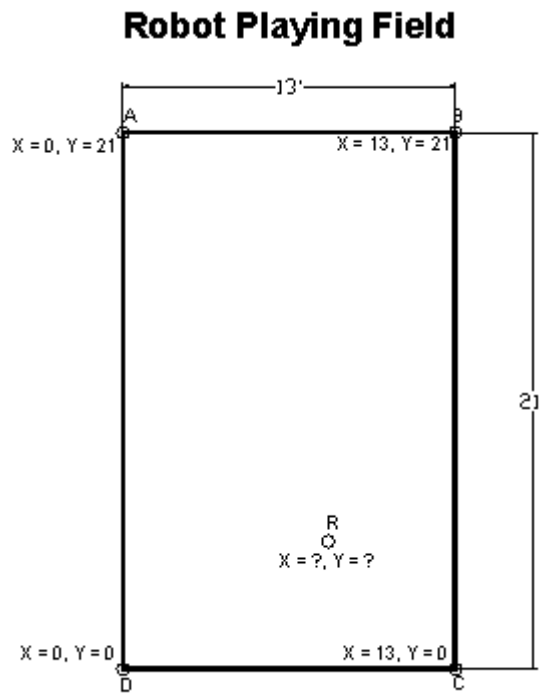
Section 3: Actual Code using the BX-24 chip and the Basic Express Development System compiler from Netmedia Inc. (Future paper) Section 3 is specific computer programming code that the author used on an actual robot to prove the implementation of the algorithm. The system described assumes the use of floating point or decimal arithmetic. Built-in trigonometry functions (sine, cosine, tangent), although not mandatory, would be a distinct advantage.

The mechanical requirements for the system are:

1. A robot with a top-mounted laser that can rotate 360° with phototransistors or other optical sensors that read the return signal from reflectors mounted at the perimeter of the playing field. The author used a standard three-wire audio plug for this function. The receptacle was mounted to the robot. The male plug had glued to it a small platform that held the laser and the phototransistors to receive the return signal. A stepper motor with a belt connected to this section rotated the laser assembly. When plugged in, the audio plug acted as a slip ring to allow full rotation of the laser. The three leads were assigned to power, ground and the signal lead to carry the received light signal to the processor.
2. This project uses three reflectors such as those in auto parts stores to be in place around the perimeter of the area. The author used red and white reflectors found in auto stores and used mostly by truckers. The reflection of an inexpensive pen laser returning to three or four phototransistors crowded around the transmitted beam typically received solid return signals up to about 25 feet.
3. A microprocessor chip capable of performing the calculations.

Example Scenario

It is the author's style to work with specifics wherever possible. The general solution is explained below for a single specific example. In the drawings to follow, Points A, B, C and D represent the northwest, northeast, southeast and southwest corners respectively, of a 13 X 21 foot, robot playing field. To keep all X,Y coordinates as positive numbers, Point D was arbitrarily chosen to be at position X=0, Y=0. Figure 1 shows the playing field coordinates.

**Figure 1**

Although the scenario will work with the playing field "pointed" in any direction, for clarity, Points A and B define the east-west line at the North boundary of the playing field.

An original playing field sized at 24 X 32 feet for another contest was to be used, however, the laser receptors degraded perceptibly past 25 feet, so the playing field was reduced so that the longest possible distance, the diagonal, was about 25 feet. Point R represents the exact position of the robot, which is unknown in the beginning. Also, Point D, the fourth corner of the rectangle is shown for viewer clarity, but it is never used in the computations or the geometry of the problem. The reflectors are intentionally placed at Points A, B and C, and NOT placed at Point D for several reasons. Picture yourself in the playing field, and rotating 360°. Since A and B are the closest together in degrees of arc, your timing should later confirm that your laser spent the least time in one sector, the northern one. Likewise, the placement at Points A and C, in your perspective, are placed farthest apart, so the laser will spend the most time scanning in the southwest sector. And of course, the eastern sector will be somewhere in between. This generally works except for a few special cases. There is of course one point where the angles between all three reflectors will all be 120 degrees. In this case, the robot would have to move, hopefully in a southwesterly direction, to resolve the problem. This algorithm also breaks down if the robot strays outside playing field, particularly if it wanders through the North or East boundary lines. For this paper, the robot is encouraged to avoid these ambiguous zones. Solving these ambiguities may well be a good topic for another paper.

From most points in the playing field, no matter which way you are originally pointing, since the laser always turns counterclockwise, you can only come up with three possible scenarios that make sense. You will sense:

Long time, medium time, short time, or

Medium time, short time, long time, or

Short time, long time, medium time

(Ignoring the single position in the playing field where all times are the same.) By recording the times between sensors, you will later be able to tell where you started out. Additionally, as we will see below, we only require three reflectors to determine an accurate position, so Point D has no reflector and is ignored.

SECTION 1

Overview of the Problem

This range-only navigation algorithm uses some geometry and trigonometry, so let's review the math. Suppose you place two reflectors 13 feet apart, at Points A and B as shown in Figure 2.

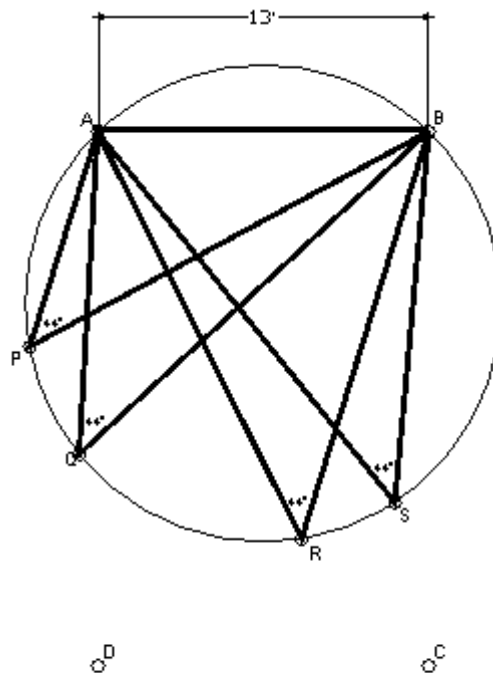


Figure 2

If the robot is at Point R, angle R of the ARB triangle can be accurately measured. At this point, one might think you can do trigonometry here on the triangle, but it won't

work. Trigonometry requires three values to solve any triangle that doesn't have a right angle of 90° . (You need to know three things; either Side/Angle/Side, Angle/Side/Angle, or Side/Side/Side for a solution.) So the trigonometry functions and the Laws of Sines and Cosines won't help here.

However, there are several geometry rules that can get us to where we want. Notice the circle in Figure 2. Geometry states that given any three points not in a straight line, there is one and only one circle that can be drawn to touch all three points. Later, I'll explain exactly how to find the center of that circle and draw it. Let's say that you measure the angle R and see that it is 44° . Another geometry rule is that no matter where you put the robot on the circle, except points A and B, the angle between points A and B will always be the same, so angles P, Q, R and S are all the same. (In our example, 44° .) So the rule now allows us to say **"If the angle from A to B is 44° , the robot MUST BE somewhere on this particular circle, but we have no idea exactly where on this circle."** But we can be sure that the robot has to be on the perimeter of this particular circle. Now, if we add a third navigation reflector, (at point C) we can use one of the existing reflectors (Point B) and the new one to give us a second circle. Using these two reflectors will produce a second circle. The robot will be located somewhere on the circumference of the second circle also, hence the exact position of the robot is where the two circles intersect.

In Figure 3, the circle for Points A, B, and R is the same as in Figure 2.

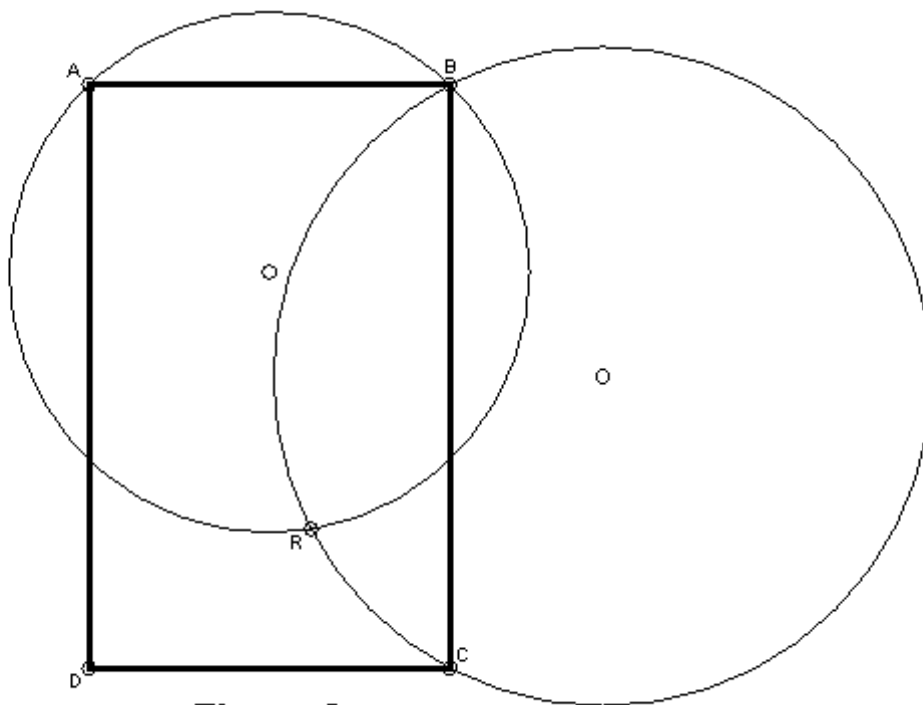


Figure 3

A second circle is drawn that passes through points B, C, and R. Since the robot must be somewhere on the circumference of Circle 1 and also somewhere on the circumference of Circle 2, the robot can only be in one of the two places where the

circles intersect, Point B or Point R. Since we know that B is the location of our reflector, we can throw out that position and accurately place our robot at Point R.

It remains but to show how to construct the circles, and then from these circles, determine point R as one of the intersection points of the circles.

And Now.... From the Robots Perspective

To get the best accuracy possible, this algorithm presumes the robot stops, then takes readings. Stopping will give the best solution. The laser could rotate while the robot is moving, however the distance traveled per each laser rotation will introduce a corresponding error in the calculations.

First, let's make the timing and angle calculations. For this task, our example laser rotates counterclockwise once every four seconds. (The exact time per rotation is unimportant as long as the speed throughout the 360° circle is constant.) The author's laser has a microswitch that "clicks" when the laser is pointed in a particular direction. For ease of computations later, it is best that the microswitch be adjusted so that it clicks just as the laser is pointing straight ahead (zero degrees relative) on the robot. Any method that lets the processor know when the laser is pointing a specific direction will work. Let's rotate the laser, and start a timer when we get the "click". Now, in a 360° rotation, there are three reflectors, so your laser should get three readings before it gets back to the "click" point. The computer might record readings like those in Table 1.

EVENT	AT TIME	TIME BETWEEN
Click	0.0000 Sec	
1 st Reflection	1.0556 Sec	1.0556 Sec
2 nd Reflection	2.3628 Sec	1.3072 Sec
3 rd Reflection	2.8508 Sec	0.4880 Sec
Click	4.0000 Sec	1.1492 Sec

Table 1: Time Measurements

Notice several things here. The time from the first to second click was four seconds. This verifies that our laser is tracking at the correct speed. A reading not considered close should generate an error or warn the operator that something is wrong. Also notice that there were three reflections in the sequence. If there were less than or more than three reflections, we know that there has been a problem. Perhaps the laser missed a reflector or it got too many readings indicating one or more false reflections.

In either case, this kind of result would show as bad to the computer which would throw out this set of readings and start over.

Also notice that the laser will usually "click" while the laser is pointing between two reflectors. To know the size of the arc involved on either side of the "click" point, we added a "TIME BETWEEN" column to Table 1. This allows us to add the time from the first click to the first reflection together with the time from the second click, backwards to the last (third) reflection, to determine the total time spent in this arc. In this example, we would add 1.0556 to $(4.000 - 2.8508) = 1.0556 + 1.1492 = 2.2048$ seconds. So the times we spent in each of the three areas, in counterclockwise order, are:

- 2.2048 Seconds Because of the placement of the reflectors, we intentionally set the reflector spacing so that the laser usually spends the longest time in the southwest sector between reflectors A and C. Since 2.2048 seconds is the longest time, we know that the robot started out pointing somewhere into the southwest sector. Later, we'll use the 0.0556 second time from the first click to the first reflection to see exactly which way the robot is pointing.
- 1.3072 Seconds This makes sense, since the time spent in the eastern sector should be the "middle" time and come just after the "longest" time.
- 0.4880 Seconds This confirms our evaluation since we expect the northern sector to have the shortest time.

Figure 4a shows where we are at this point.

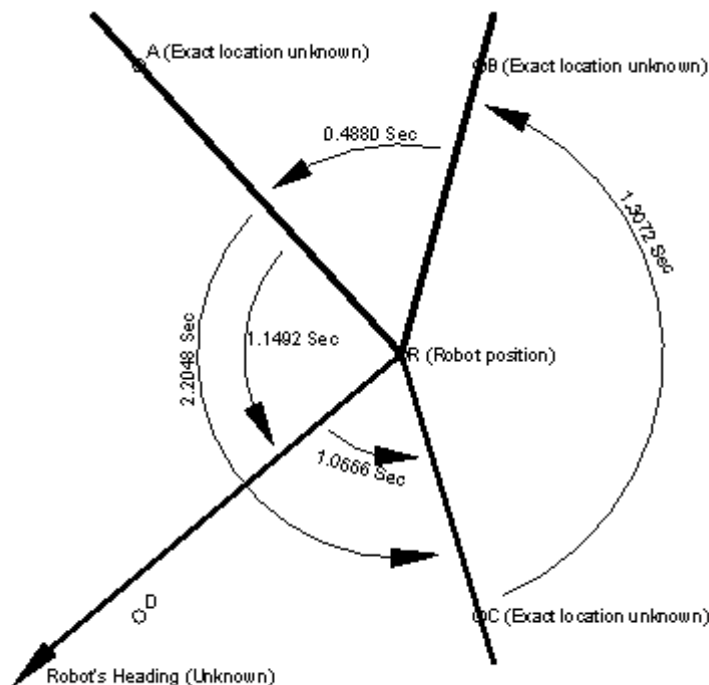


Figure 4a

Remember, that at this point, although we can draw a nice neat Figure 4a, the robot has no idea about distance. Only the times and the ability to know which times represent which sector.

Next, we can compute the interior angles. Since the time spent in each sector is in direct proportion the angle of the arc scanned, and 4.000 seconds equates to 360° , we use the formula:

Sector Angle = Time in Sector divided by Total Time times 360°

Note: If the rotation time was constant and never varied, total time would always be 4 seconds, and since it never changes, we could simplify the equation to:

$$\text{Sector Angle} = \text{Sector time} \times 120$$

But, since we want to allow the computer to work with values that are close to, but not always exactly 4 seconds, we chose to read and store the total time for each revolution.

Applying the formula to each arc time gives:

$$2.2048 \text{ sec} / 4 \text{ sec} \times 360^\circ = 198.4320^\circ$$

$$1.3072 \text{ sec} / 4 \text{ sec} \times 360^\circ = 117.6480^\circ$$

$$0.4880 \text{ sec} / 4 \text{ sec} \times 360^\circ = 43.9200^\circ$$

Total should be: 360.0000°

The situation at this point is illustrated in Figure 4b.

You may notice that the line for the robot's heading has been removed. That is because we won't need this for a while. Later, when we know the robot's exact position, we'll come back to this to compute the direction the robot is heading.

We don't need the 198.4320° angle for our following computations, but it is handy if we want to double check that all three angles add up to 360° before we throw it out. Now we are ready to begin finding our circle centers.

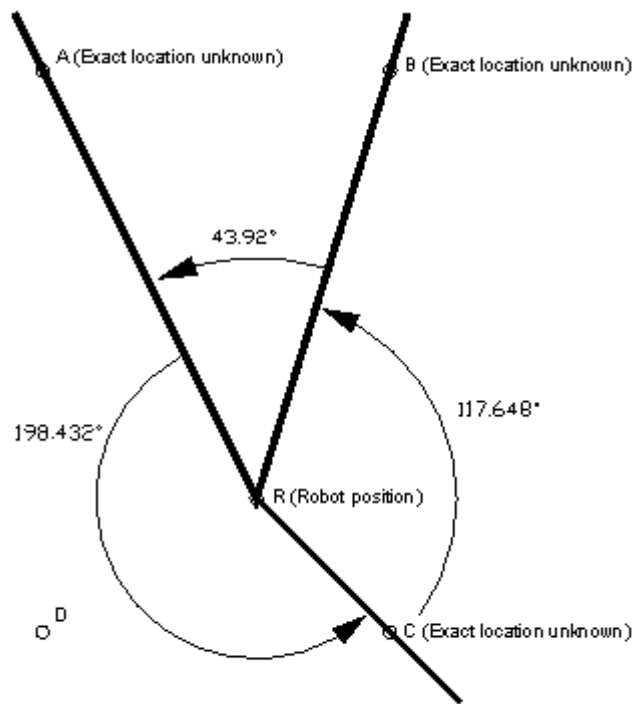


Figure 4b

From Figures 4a and 4b, we know that Points A, B and C are somewhere along the lines drawn, but we can't tell where exactly they should be drawn, so we will come from the other direction in Figure 5.

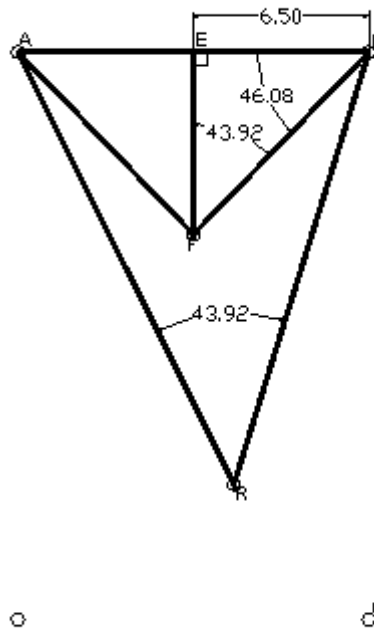


Figure 5

Initially, in Figure 5, we know is the distance between Points A and B is 13 feet and that the ARB angle is 43.9200° . Eventually, A and B and R will all be points on the circumference of the circle we will construct. Because of this, angle R becomes what is

called an inscribed angle, and the geometry inscribed angle rule states that the angle from the center of the circle to points A and B will be twice angle R (43.920°) or 87.840° . Create this angle as follows:

1. From the midpoint of line AB drop a perpendicular line down. Since the length is known to be 13 feet, the midpoint is at 6.5 feet.
2. Construct a triangle such that the interior angle BFE is 43.920° . (Also shown is triangle AFE, which is a mirror image of triangle BFE, and since we only need to solve one of the two triangles [BFE], we can ignore the AFE triangle.)

Since we bisected line AB with the perpendicular line DE, we also know we have a right triangle and that the lengths of the side BE is half of line AB or $13 \text{ feet} / 2 = 6.5 \text{ feet}$. Also, since we have a right triangle, the angle EBF is known to be $90^\circ - 43.920^\circ$ or 46.080° .

Solving the BFE triangle in Figure 5 will tell us two important things. First, Point F is the center of the circle we are looking for. This requires us to find the length of line EF so that we know exactly how far down from Point E we will find Point F. Second, we need to know the length of the hypotenuse of triangle BFE (line BF) because this line becomes the radius of the circle we will draw. Or - - - **A circle, drawn from point F with a radius FB, will pass through Points A, B and R.**

So we solve the triangle for those values. For the adjacent line DE:

$$\text{Tangent EFB} = \text{Opposite} / \text{Adjacent}$$

Transposing:

$$\text{Adjacent} = \text{Opposite} / \tan 43.92^\circ$$

$$= 6.5 \text{ feet} / 0.963$$

$$= 6.750 \text{ feet}$$

Note: Without trig functions, you can solve the triangle by using the Pythagorean Theorem where the hypotenuse squared equals the sum of the squares of the opposite and adjacent sides. Or.....

$$c^2 = a^2 + b^2$$

For the hypotenuse line BF (The radius of the circle):

$$\sin BDE = \text{Opposite} / \text{Hypotenuse}$$

Transposing:

$$\text{Hypotenuse} = \text{Opposite} / \sin 43.92^\circ$$

$$= 6.5 \text{ feet} / 0.694$$

$$= 9.371 \text{ feet}$$

So, the center of the first circle is 6.5 feet to the right of Point A ($X = 6.5$ feet) and 6.750 feet below line AB. ($Y = 21 - 6.750 = 14.250$ feet) All we have to do now, is draw a circle from point F ($x = 6.5$, $y = 14.25$) with a radius of 9.371 feet and the circle will pass through the three points A, B, and R. The completed portion for our first circle is shown in Figure 6 along with the new dimensions we have computed.

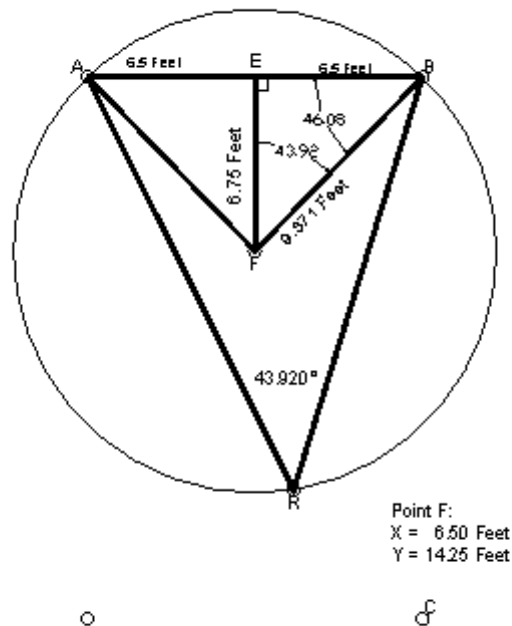


Figure 6

At this point, we know the robot is somewhere on this circle. Next, we are going to do the same type of computations between Points B and C to develop the second circle. After that, we will solve the two circles for their points in common.

The Second Circle

The next triangle to solve is triangle BRC that was first shown in Figure 3. There will be one difference you should note about our second circle. In Figure 3, because the interior angle of the ARB inscribed angle was less than 90° , the circle center wound up on the near side of or below the line AB. If that angle had been greater than 90° , the circle center would have wound up on the far side of or above the line AB. In Figure 3,

since angle BRC is greater than 90° (117.648°), the circle center will wind up on the far (in this case the right) side of the line BC. Now shift your attention to Figure 7.

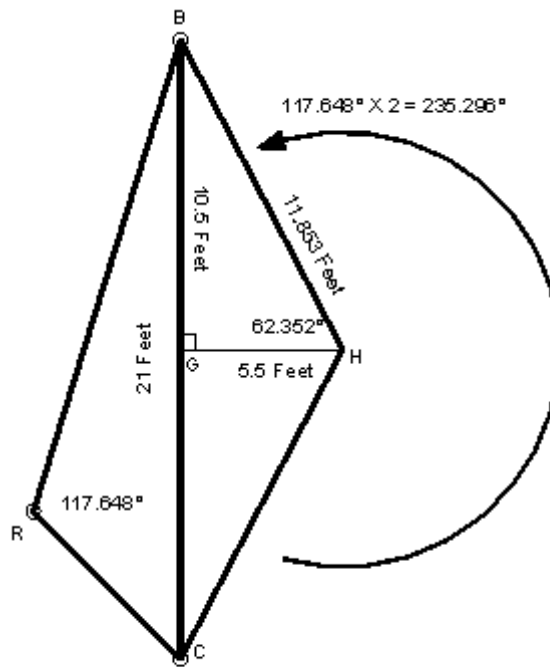


Figure 7

Notice that to get twice the 117.648° angle, we have to construct it on the far side of line BC and we wind up with an exterior angle of $117.648^\circ \times 2$ or 235.296° at Point F, as shown in Figure 7. To get our triangle to solve, we know that the angle BHG is 360° minus the exterior angle, 235.296° divided by 2 or 62.352° and we construct the BHG triangle. After construction, because we did a perpendicular bisection of line BC, angle BGH is a right angle, or 90° . We now have enough information to solve triangle BGH.

Similar to what we did before, we are looking for the length of line GH to tell us how far to the right of line BC the circle center is and for the radius of the circle which is the hypotenuse line BH.

$$\text{Tangent BHG} = \text{Opposite} / \text{Adjacent}$$

Transposing...

$$\text{Adjacent} = \text{Opposite} / \text{Tangent BHG}$$

$$= 10.5 \text{ feet} / \text{Tangent } 62.352^\circ$$

$$= 10.5 \text{ feet} / 1.909$$

$$= 5.500 \text{ feet}$$

$$\text{Sine BHG} = \text{Opposite} / \text{Hypotenuse}$$

Transposing...

$$\text{Hypotenuse} = \text{Opposite} / \sin \text{BHG}$$

$$= 10.5 \text{ feet} / \sin 62.352^\circ$$

$$= 10.5 \text{ feet} / 0.886$$

$$= 11.8535 \text{ feet}$$

So, our second circle is to be drawn 5.500 feet to the right of line BC, making

$$x = 13 + 5.5$$

$$= 18.5 \text{ feet}$$

and half-way between B and C or

$$y = 21 \text{ feet} / 2$$

$$= 10.5 \text{ feet.}$$

And a circle drawn from point $x = 18.5 \text{ feet}$, $y = 10.5 \text{ feet}$, with a radius of 11.8535 feet will pass through points B, R and C.

Putting the Circles Together

Figure 8 now has both circles we have constructed.

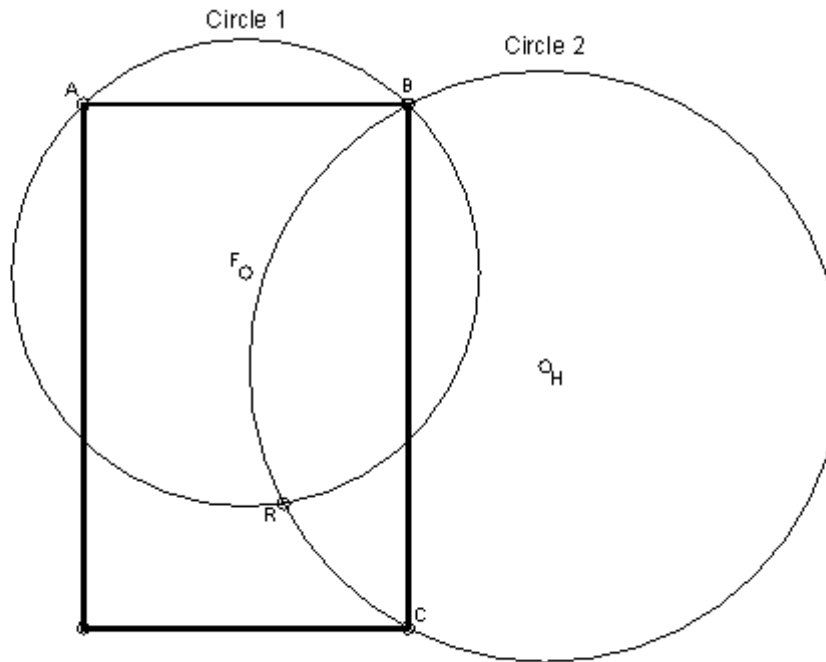


Figure 8

Notice that Circle 1 and Circle 2 intersect at two points. The first is at $x = 13$, $y = 21$, or Point B, and the second intersection is at Point R. Shortly, we will solve the two circles to get the exact location of Point R.

Let's review and consolidate at this point and take everything we know and apply this information to Figure 8. We will also include the X-Y coordinates of the playing field since we ultimately want to know the X-Y coordinates of our robot in relation to our playing field.

Point D was originally selected to be $X=0$ and $Y=0$ so that X and Y coordinates on the playing field will be positive numbers.

Points A, B, C and D are the corners of the playing field. Point F, the center of Circle 1, is half the distance between A and B which is known to be 13 feet long, so $X = 6.5$. Y is the height of the playing field, 21 feet, minus the length of the side of the small triangle, 6.75 feet, or $Y = 14.25$. Point G, the center of Circle 2, is half the distance between B and C which is known to be 21 feet long, so $Y = 10.5$ feet. X is the width of the rectangle, 13 feet, plus the length of the side of the small triangle, 5.5 feet, or $X = 18.5$ feet.

Here is all of the data we have developed so far:

Point	X	Y
A	0	21
B	13	21

C	13	0	
D	0	0	
F	6.5	14.25	(The center of Circle 1)
G	18.5	10.5	(The center of Circle 2)
R	?	?	(The location of the robot. Still unknown!)

...and we also know...

Radius of Circle 1 = 9.371 feet

Radius of Circle 2 = 11.8535 feet

Finally! On to the Circles.

Once we know the location of the two circles, we can use the general equation for a circle, which is

$$(X - H)^2 + (Y - K)^2 = R^2$$

You may recognize the shorter form $X^2 + Y^2 = R^2$ as the Pythagorean Theorem which works when the circle is centered at 0,0. We'll use the H and K as the offsets to the circle centers and plug in our known values to look for X and Y. If we fill in the information for Circle 1 in the equation, and the information for Circle 2 in an identical equation, we will wind up with two equations with two unknowns which we attack with the algebra rules for solving simultaneous equations.

Circle 1: $(X - H)^2 + (Y - K)^2 = R^2$

$$(X - 6.5)^2 + (Y - 14.25)^2 = 9.371^2$$

$$X^2 - 13X + 42.25 + Y^2 - 28.5Y + 203.0625 = 87.8156$$

Simplify:

$$X^2 - 13X + Y^2 - 28.5Y + 157.4969 = 0 \text{ (Equation \#1)}$$

Circle 2: $(X - H)^2 + (Y - K)^2 = R^2$

$$(X - 18.5)^2 + (Y - 10.5)^2 = 11.8535^2$$

$$X^2 - 37X + 342.25 + Y^2 - 21Y + 110.25 = 140.5055$$

Simplify:

$$X^2 - 37X + Y^2 - 21Y + 311.9945 = 0 \text{ (Equation \#2)}$$

Now subtract Equation #1 from Equation #2:

$$\text{(Equation \#2)} \quad X^2 - 37X + Y^2 - 21.0Y + 311.9945 = 0$$

$$\text{(Equation \#1)} \quad -X^2 + 13X - Y^2 + 28.5Y - 157.4969 = 0$$

$$\text{(Equation \#3)} \quad -24X + 7.5Y + 154.4976 = 0$$

And solve for X:

$$-24X = -7.5Y - 154.4976$$

Divide by -24:

$$X = 0.3125Y + 6.4374$$

Substitute this value for X into Equation #1:

$$X^2 - 13X + Y^2 - 28.5Y + 157.4969 = 0$$

$$(0.3125Y + 6.4374)^2 - 13(0.3125Y + 6.4374) + Y^2 - 28.5Y + 157.4969 = 0$$

Multiply:

$$0.0977Y^2 + 4.0234Y + 41.4401 - 4.0625Y - 83.6862 + Y^2 - 28.5Y + 157.4969 = 0$$

Combine like terms:

$$1.0977Y^2 - 28.5391Y + 115.2508 = 0$$

Divide by 1.0977:

$$Y^2 - 25.9990Y + 104.9930 = 0$$

Solve by completing the square: (Move the number to the right:)

$$Y^2 - 25.9990Y = -104.9930$$

Make the left side a square by taking half of the coefficient of Y and square it

$$\begin{aligned} & (-25.9990 / 2)^2 \\ &= -12.9995^2 \\ &= 168.9870 \end{aligned}$$

and add that to both sides:

$$Y^2 - 25.9990Y + 168.9870 = -104.9930 + 168.9870$$

Or

$$(Y - 13)^2 = 64$$

Take square root of both sides:

$$Y - 13 = \pm 8$$

$$Y = 13 + 8$$

$$= 21 \text{ and also...}$$

$$Y = 13 - 8$$

$$= 5$$

So, the 2 values for Y are 21 and 5. (The computer doesn't know it yet, but we can see that 21 is the Y value for Points A and B, so 5 must be the Y value for the robot's location!)

Substitute each of these values in Equation #3:

For Y = 21:

$$-24X + 7.5Y + 154.4976 = 0$$

$$-24X + 7.5(21) + 154.4976 = 0$$

$$-24X + 157.5 + 154.4976 = 0$$

$$-24X + 311.9976 = 0$$

$$-24X = -311.9976$$

$$X = 12.999 \text{ Feet} = 13 \text{ Feet}$$

Since $X = 13$, $Y=21$ is the location of Point B, we can ignore this and get the location of the robot next.

For $Y = 5$:

$$-24X + 7.5Y + 154.4976 = 0$$

$$-24X + 7.5(5) + 154.4976 = 0$$

$$-24X + 37.5 + 154.4976 = 0$$

$$-24X + 191.9976 = 0$$

$$-24X = -191.9976$$

$$X = 7.999 \text{ Feet}$$

$$= 8 \text{ Feet}$$

So, FINALLY..... we find the robot at

$$X = 8, Y = 5.$$

Which Way is the Robot Heading?

Finding out where we are is a great achievement, but we need to go a few steps further. Is the robot in its final resting place? Fine if it is, but most of the time it won't be, so we need to know three important things.

1. In what direction is my next (or final) destination?
2. How far away is it? ...and...
3. Which way and how far do I turn to get headed in the right direction? Or which way am I pointing now?

For illustration, let's say that there is a power receptacle in the middle of the west wall of the playing field, and we want the robot to return "home" and recharge batteries. Refer to Figure 9 and let's set up what we know.

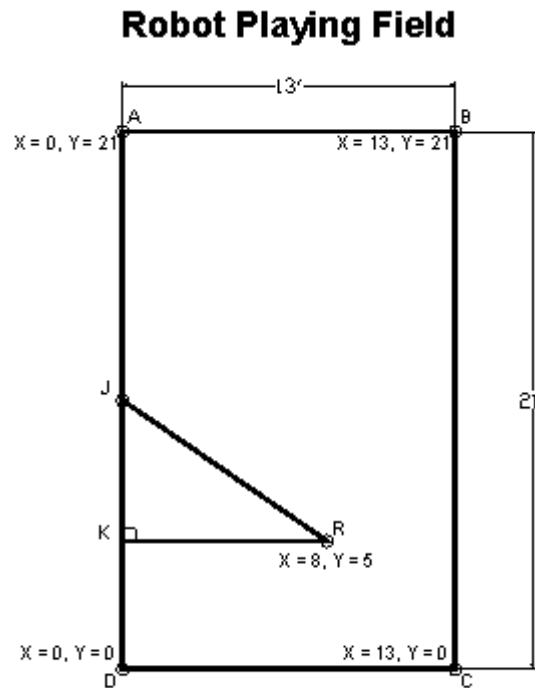


Figure 9

We know where the robot is located:

$$X = 8, Y = 5$$

We know the plug is in the middle of the west wall that is 21 feet long so it is at:

$$X = 0 \text{ feet}$$

$$Y = 21 / 2$$

$$= 10.5 \text{ feet}$$

There are several easy ways to find the length of the line JR. One is the Distance Formula (Just another form of the Pythagorean Theorem) which is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(0 - 8)^2 + (10.5 - 5)^2}$$

$$d = \sqrt{-8^2 + 5.5^2}$$

$$d = \sqrt{64 + 30.25}$$

$$d = \sqrt{94.25}$$

$$d = 9.5 \text{ feet}$$

So 9.5 feet is the distance between Points R and J.

To find out the direction to Point J, we can use trig to solve for angle R. Since we know both the opposite side length and the adjacent side length, we can use:

$$\tan R = \text{Opposite} / \text{Adjacent}$$

$$\tan R = 5.5 \text{ feet} / 8 \text{ feet}$$

$$\tan R = 0.6875$$

$$\text{Angle } R = 34.5085^\circ$$

Since robots typically can't steer to this accuracy, 35° will be close enough. With 0° being North and 270° being West, we need to go 35° to the right of 270° or steer a course of 305° for 9.5 feet.

One More Thing!

Last, but certainly not least is to compute how far and in what direction to turn to be headed in the right direction. Remember way.... Way.... Back... just after Figure 1, I told you we were going to use the time between the first click and the first reflection (1.0556 seconds) to determine which way the robot was pointing? Well, here is where we do that. Go back and take a look at Figure 7 for a moment. We didn't know it then, but we now know the coordinates of Points R and C are:

	X	Y
R	8	5
C	13	0

To get from Point R to point C we would have to travel 5 feet to the right and 5 feet down, and that means that angle BCR in Figure 7 is 45° exactly. If the robot wanted to go to Point C it would travel southeast or on a heading of 135° . Since the laser took 1.0556 seconds to get pointed in this direction, and it travels 360° in 4 seconds, we can do a proportion problem:

1.0556 seconds is to w seconds as 4 seconds is to 360° , or

$$w^\circ / 360^\circ = 1.0556 / 4$$

Cross-multiplying gives:

$$4w = 360(1.0556)$$

$$4w = 380.016$$

$$w = 95.004$$

So we know the robot started out pointed 95° from Point C, so $135^\circ + 95^\circ = 230^\circ$. So the robot is pointing 230° and it wants to steer a course of 305° , so it needs to go $305^\circ - 230^\circ$ or make a right turn of 75° before it takes off.

You may have noticed a flaw above. We used the coincidence that the line RC was exactly at 45° , but in reality, that is almost never the case, and anyway, we have to provide the math to allow a solution no matter what the angle or position. The solution requires we solve the triangle BCR in Figure 7 to come up with angle BCR. We know the length of the long side to be 21 feet and the angle BRC is 117.648° , so we only need one more known to be able to apply the law of sines. (We assume that no angle of this triangle is ever exactly 90° , since that is highly unlikely, and since it almost never is, we need to depend on either the Law of Sines or the Law of Cosines when we do not have a right angle. With R at $X = 8$, $Y = 5$, and C at $X = 13$, $Y = 0$, we can again use the Pythagorean Theorem to solve line RC.

$$c^2 = a^2 + b^2$$

$$\text{Line RC}^2 = 5^2 + 5^2$$

$$RC^2 = 25 + 25$$

$$RC^2 = 50$$

$$\text{—————}$$

$$RC = \sqrt{50}$$

$$RC = 7.071 \text{ feet}$$

The Law of Sines is:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{21}{\sin 117.648^\circ} = \frac{c}{\sin C} = \frac{7.071}{\sin B}$$

Since we didn't learn anything about C, we can throw the center term out and work the proportion:

$$\frac{21}{\sin 117.648^\circ} = \frac{7.071}{\sin B}$$

Cross multiply:

$$21(\sin B) = 7.071(\sin 117.648^\circ)$$

$$21(\sin B) = 7.071(0.8858)$$

$$21(\sin B) = 6.2636$$

$$\sin B = 6.2636 / 21$$

$$\sin B = 0.2983$$

$$\text{Angle } B = 17.3555^\circ$$

Since the interior angles of a triangle always add up to 180° ;

$$\text{Angle } C = 180^\circ - 17.3555^\circ - 117.648^\circ$$

$$\text{Angle } C = 44.9965^\circ$$

Or an angle of 45° as we suspected. This confirms that the robot will have to make a right turn of 75° to proceed in the correct direction.

Summary

So, it is possible to navigate a small autonomous robot using passive reflectors. There is yet work to be done. Is it possible to use the algorithm to navigate outside of the playing field? In his work cited below, Jim Ubersetzig successfully used three reflectors along a wall for accurate positioning. But if the reflectors were free standing (no wall), then would the robot wander past the plane of the reflectors and become "lost" when on the wrong side of the reflectors? Is there a possible placement of three or more sensors that will allow resolution of the ambiguities near corners and boundary lines? The author hopes to pursue these and other navigation problems as this project matures.

Inspiration and Resources

The following sources gave me the ideas and inspiration for this project. If you have any additional resources relative to this algorithm, please let me know at www.trainprogrammers.com or richardv2@home.com.

(Borenstein, Everett and Feng) J. Borenstein, H. R. Everett, L. Feng. *Where Am I?: Sensors and Methods for Mobile Robot Positioning*, University of Michigan, 1996 (CD-ROM with many robotics papers)

(Borenstein, Everett, Feng and Wehe) J. Borenstein, H. R. Everett, L. Feng, D. Wehe. *Mobile Robot Positioning – Sensors and Techniques*, Journal of Robotic Systems, September 1996

(Everett) H. R. Everett. *Sensors for Mobile Robots: Theory and Application*, A. K. Peters Ltd., 1995

(Jones, Seiger and Flynn) Joseph L. Jones, Bruce A. Seiger and Anita M. Flynn. *Mobile Robots: Inspiration to Implementation, Second Edition*, A.K. Peters, Natick, MA, 1999

(McComb) Gordon McComb. *Robot Builder's Bonanza: 99 Inexpensive Robotics Projects*, Tab Books, 1987, Chapter 29, Navigating Through Space

(Miller, Winkless, Botsworth and Phillips) Merl K. Miller, Nelson B. Winkless, Joseph H. Bosworth, Kent Phillips. *The Personal Robot Navigator*, A. K/ Peters, Ltd., 1998

(Ubersetzig) Jim Ubersetzig. *A Circular Navigation System*. The Robot Builder Newsletter for the Robotics Society of Southern California, Sep, Oct, Nov 1999, <http://www.dreamdroid.com/newsletter.htm>

www.smartrobots.com, Schwartz Electro-Optics in Orlando Florida, Intelligent Solutions, Inc. in Marblehead MA, Denning Branch Mobile Robotics in Pittsburgh PA and Guidance Control Systems in Leicester (England?) all advertise laser reflector navigation kits for prices from \$445.00 to \$12,000.00. There are probably others. Although none of these companies reveal the algorithm used, it appears they use an algorithm similar to or exactly the same as that described in this paper. I lay no claim to originality, original invention, patent or other rights. I do, however, cheerfully reveal that I built the system described for under \$30.00!